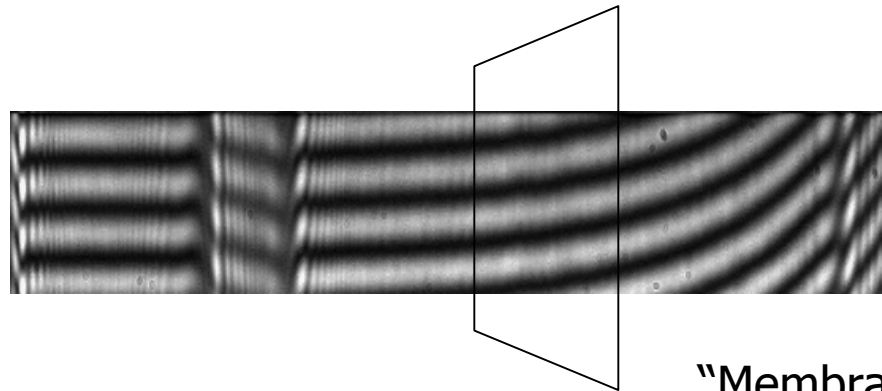


Extracting thermodynamic parameters from sedimentation equilibrium data

BBRI 2008

Sedimentation Equilibrium: osmotic pressure description



Osmotic pressure of i^{th} component across the membrane

$$\frac{\pi}{c_i RT} = \frac{1}{M_i} + A_2 c_i + \dots$$

$$\left[\frac{d\pi_i}{dc_i} \right] = \frac{1}{M_i} + 2A_2 c_i = \frac{1}{M_i} \left\{ 1 + \frac{d \ln \gamma_i}{d \ln c} \right\}$$

Multiple, non-interacting components

$$\frac{\pi}{c_i RT} = \frac{1}{M_i} + A_2 c_i + \dots$$

$$\left[\frac{d\pi_i}{dc_i} \right] = \frac{1}{M_i} + 2A_2 c_i = \frac{1}{M_i} \left\{ 1 + \frac{d \ln \gamma_i}{d \ln c} \right\}$$

$$\sigma_i \equiv \frac{d \ln c_i}{d \frac{r^2}{2}} = \frac{M_i (1 - \bar{v} \rho) \omega^2}{RT \left(1 + \frac{d \ln \gamma_i}{d \ln c} \right)} \cong \frac{\omega^2 \frac{d\rho}{dc_i}}{\frac{d\pi_i}{dc_i}}$$

Approximate dp/dc
Note that nonideality
uses c not c_i

$$c_{r,i} = c_{o,i} e^{\sigma_i \left(\frac{r^2 - r_0^2}{2} \right)}$$

Some solution scenarios

Multiple independent components

$$c_r = \sum c_{r,i} = \sum c_{o,i} e^{\sigma_i \left(\frac{r^2 - r_o^2}{2} \right)}$$

One component, multiple species linked by mass-action equilibria

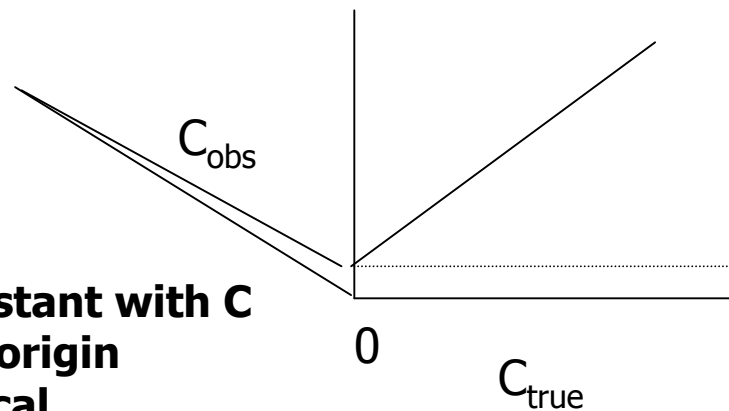
$$c_r = \sum c_{r,i} = c_{r,1} + K_i (c_{r,1})$$

For repulsive nonideality

$$\sigma_{i,\text{apparent}} = \frac{\sigma_i}{1 + \frac{\partial \ln \gamma_i}{\partial \ln c}} \cong \frac{\sigma_i}{1 + 2B_1 c + 3B_2 c^2 + \dots}$$

Dealing with baseline offset

$$C_{\text{obs}} = C_{\text{true}} + _$$



- **The offset is assumed to be constant with C**
- **Sometimes $_$ is instrumental in origin**
 - **Electrical, optical, mechanical**
- **Sometimes $_$ is chemical in origin**
 - **BME or DTT oxidation**
- **There may be uncertainty in the value of $_$**
 - **Included as a fitting parameter**

Dealing with $M_{-}/Mc_i \cong (1-v\rho)$ in an associating system

- Assume that the change in volume with association or with pressure is small
- Approximate differences may be lifted by allowing non-integral values of the stoichiometry, i

$$c_r = \sum c_{r,i} = c_{r,1} + K_i (c_{r,1})^{I(i)}$$
$$I(i) = \frac{i(1 - \bar{v}\rho)_i}{(1 - \bar{v}\rho)_1}$$

Dealing with cross-nonideal terms

- λ $M \ln \frac{c_i}{M c}$ is an approximation since each of the solution components will give rise to separate terms of the form $M \ln \frac{c_i}{M c_j}$ where $i \neq j$ and $i = j$ contribute
- λ For associating systems, the Adams-Fujita approximation is made
- λ $n \ln \frac{c_m}{c_n} = \ln \frac{c_m}{c_n}$, where m is monomer, and n is the n -mer

Fitting function actually used

$$F = \sum_{i=1}^n \left\{ \left(e^{\ln c_{o,1}} \right) \exp \left[\frac{\ln K_i}{I(i)} + \sigma_1 \left(\frac{r^2 - r_o^2}{2} \right) - \sum_{k=1}^m \frac{k+1}{k} B_k (F - \delta)^k \right] \right\}^{I(i)} + \delta$$

$$I(i) = \frac{i(1 - \bar{v}\rho)_i}{(1 - \bar{v}\rho)_1}$$

$$K_1 = 1$$

$c_{o,1}$ actually evaluated as $e^{\ln c_{o,1}}$ to ensure $c > 0$

Notice that F appears on both LHS and RHS of the equation

Global versus local parameters

- One of the useful features of curve fitting is that you can combine data from several experiments
- This gives rise to the concept of local variables (fit within a data set) and global variables (same value must be obtained for all data sets)

Will have j functions, F_j , one for each data set

$$F = \sum_{i=1}^n \left\{ \left(e^{\ln c_{o,1}} \right) \exp \left[\frac{\ln K_i}{I(i)} + \sigma_1 \left(\frac{r^2 - r_o^2}{2} \right) - \sum_{k=1}^m \frac{k+1}{k} B_k (F - \delta)^k \right] \right\}^{I(i)} + \delta$$

Local and global parameters

Local

$$F_j = \sum_{i=1}^n \left\{ \left(e^{\ln c_{o,1}} \right) \exp \left[\frac{\ln K_i}{I(i)} + \sigma_1 \left(\frac{r^2 - r_o^2}{2} \right) - \sum_{k=1}^m \frac{k+1}{k} B_k (F_j - \delta)^k \right] \right\} + \delta$$

Global

Dealing with data acquired at different rotor speeds

$$F_j = \sum_{i=1}^n \left\{ \left(e^{\ln c_{o,1}} \right) \exp \left[\frac{\ln K_i}{I(i)} + W_j \sigma_1 \left(\frac{r^2 - r_o^2}{2} \right) - \sum_{k=1}^m \frac{k+1}{k} B_k (F_j - \delta)^k \right] \right\}^{I(i)} + \delta$$

$$W_j = \left(\frac{\text{rpm}_j}{\text{rpm}_1} \right)^2$$

Dealing with different concentration scales

$$F_j = A_j \sum_{i=1}^n \left\{ \left(e^{\ln c_{o,1}} \right) \exp \left[\frac{\ln K_i}{I(i)} + W_j \sigma_1 \left(\frac{r^2 - r_o^2}{2} \right) - \sum_{k=1}^m \frac{k+1}{k} B_k \left(\frac{F_j}{A_j} - \delta \right)^k \right] \right\}^{I(i)} + \frac{\delta}{A_j}$$

A_j linearly scales the concentrations. This is useful for combining data acquired at different wavelengths, or using different pathlength cells or using different optical systems.

Self association versus a hetero-association

~~A hetero-association has multiple components and multiple species~~

$$\begin{aligned}
 c_c(r) &= \sum_i c_i(r) \\
 &= c_{o1} e^{\sigma_1 \xi} + \sum_m c_{o1}^m K'_{a1 \leftrightarrow m} e^{m \sigma_1 \xi} \quad \begin{array}{l} \text{self association of component 1} \\ \text{self association of component 1} \end{array} \\
 &\quad + c_{o2} e^{\sigma_2 \xi} + \sum_n c_{o2}^n K''_{a1 \leftrightarrow n} e^{n \sigma_2 \xi} \quad \text{self association of component 2} \\
 &\quad + \sum_{j,k} c_{o1}^j c_{o2}^k K'''_{a1 \leftrightarrow j+k} e^{(j \sigma_1 + k \sigma_2) \xi} \quad \text{heteroassociation}
 \end{aligned}$$

One component
m species

Must conduct separate experiments on each component alone

$$\begin{aligned}
 c(r) &= \sum c_i(r) \\
 &= c_{o1} e^{\sigma_1 \xi} + \sum_m c_{o1}^m K'_{a1 \Leftrightarrow m} e^{m \sigma_1 \xi} && \text{self association of component 1} \\
 &+ c_{o2} e^{\sigma_2 \xi} + \sum_n c_{o2}^n K''_{a1 \Leftrightarrow n} e^{n \sigma_2 \xi} && \text{self association of component 2} \\
 &+ \sum_{j,k} c_{o1}^j c_{o2}^k K'''_{a1 \Leftrightarrow j+k} e^{(j \sigma_1 + k \sigma_2) \xi} && \text{heteroassociation}
 \end{aligned}$$

The global parameters in these two lines *must* be the same in the mixture

Only need to fit for the parameters in this line. Assumes B negligible.

Golden rules about studying hetero associations

- Must characterize the thermodynamics of each component alone
- Data must be acquired at 3 or more loading concentrations
 - Span ~ 1 log in the loading concentrations
 - Same total concentration range should be used for the individual components and the mixtures
- Data should be acquired at different mole ratios of the components